

Of colored glass and saturation scales

A gluonic medium encountered in modern high energy collider experiments



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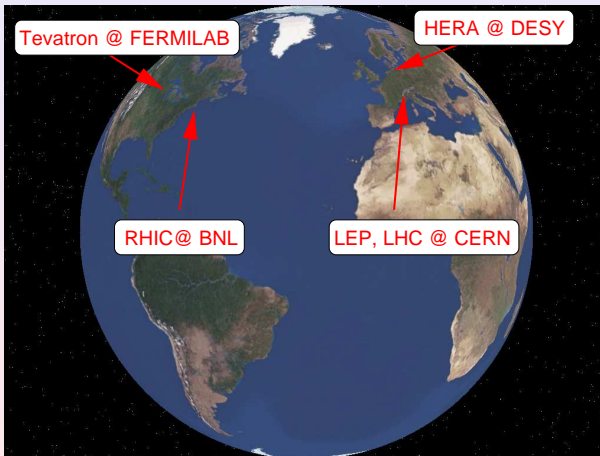
Outline

- 1 Motivation: gluons form the CGC
 - Current and planned collider experiments
 - Enhanced gluon production at high energies
 - CGC: why the name
- 2 JIMWLK evolution: properties of the CGC
 - Gluons in observables
 - The evolution equation
 - The saturation scale
- 3 Experiment
 - Geometric scaling @ HERA
 - Erasing the Cronin effect @ RHIC
- 4 Overview and outlook

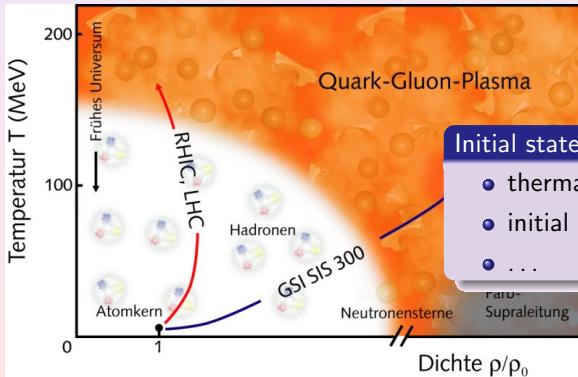
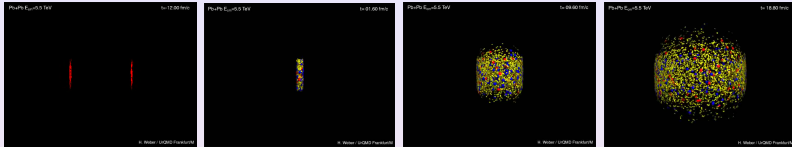
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Current collider experiments worldwide



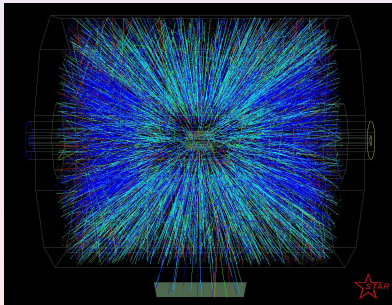
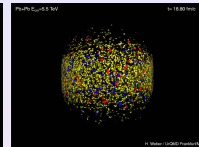
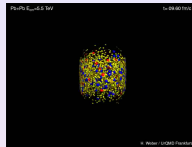
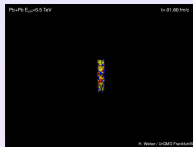
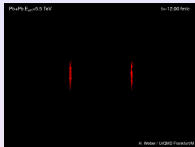
RHIC: searching for the Quark Gluon Plasma



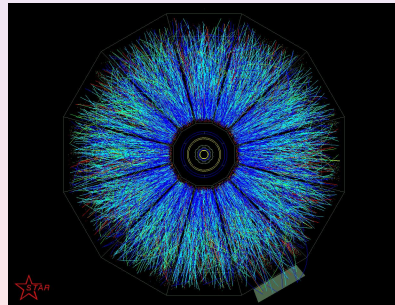
Initial state determines final state via

- thermalization times
- initial hard particle production
- ...

RHIC: searching for the Quark Gluon Plasma



side view



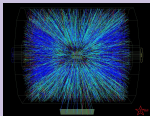
front view

Particle production at modern colliders

Large amounts of energy available: 200-14000 m_{proton}

heavy ions @ RHIC

- 200GeV/nucleon pair QGP



particle & heavy ions @ LHC

- 4TeV/nucleon pair QGP
- 14TeV to cross production thresholds Higgs

amount of data/event \equiv capacity of phone lines in Europe

QCD drives particle production

- serious backgrounds for particle searches
- new physics phenomena

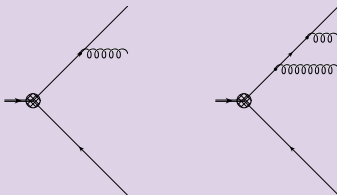
copious gluon
production



Color Glass Condensate
CGC

From photons to gluons

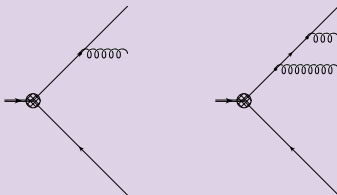
photon-like contributions



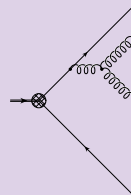
- enhanced by phase space integrals $\frac{dE}{E} \frac{d\theta}{\theta} \rightarrow \alpha_s \ln E \ln \theta$
- all orders calculation needed $\sum_{n=0}^{\infty} (\alpha_s \ln E)^n \dots$

From photons to gluons

photon-like contributions

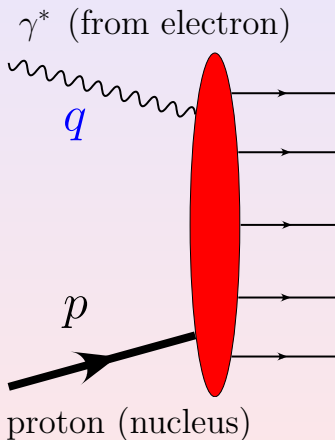


QCD: **charged** gluons



- **enhanced** by phase space integrals $\frac{dE}{E} \frac{d\theta}{\theta} \rightarrow \alpha_s \ln E \ln \theta$
- all orders calculation needed $\sum_{n=0}^{\infty} (\alpha_s \ln E)^n \dots$
- gluons **charged** \rightarrow radiation **nonlinear** in QCD

Kinematic variables: transverse resolution vs energy



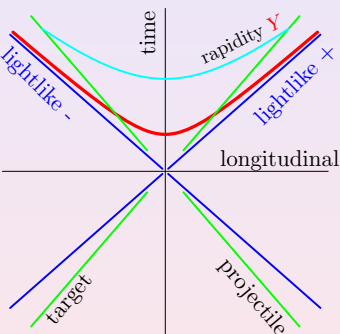
- $Q^2 := -q^2 \gg 0$

spacelike!

transverse resolution

$$\Delta r \sim \frac{1}{Q}$$

- $x = x_{\text{Bj}} := \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2m E_{\text{rest}}}$



- $Q^2 := -q^2 \gg 0$

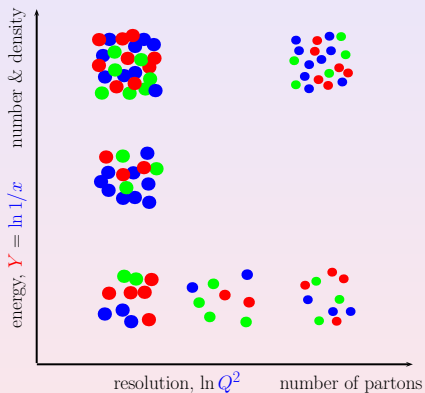
spacelike!

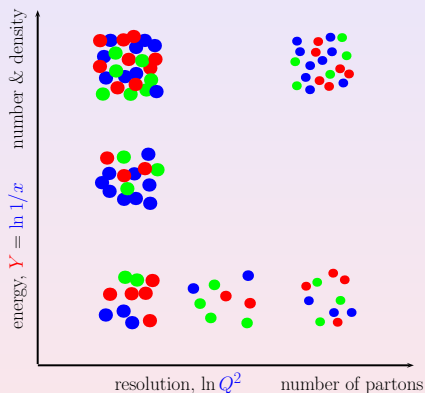
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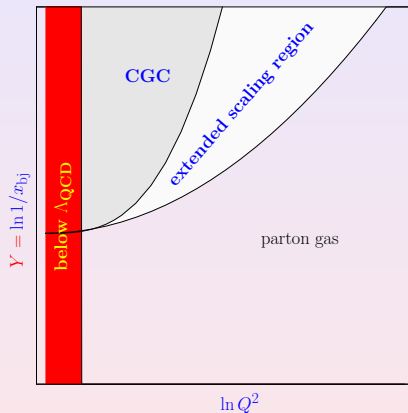
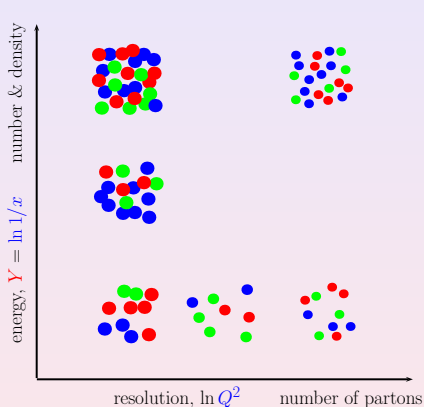
- $Y = \ln \frac{1}{x} \propto \ln E_{\text{rest}}$ all used synonymously



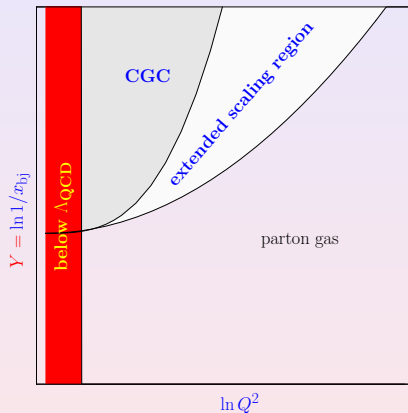


- density \rightarrow finite correlation length R_s

Large energies mean large densities



- density \rightarrow finite correlation length R_s



- Why CGC?

Why the name?

Color **Glass** **Condensate**

QCD

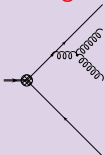
fields evolve slowly
relative to
natural scales

phase space density
 $\sim 1/\alpha_s$ & saturates

[nothing more specific implied!]

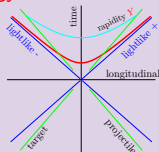
QCD

quarks and gluons



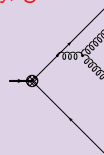
time scales

energy, time dilation



density

energy, gluons charged

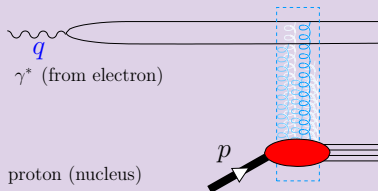


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Gluon production at increasing energy

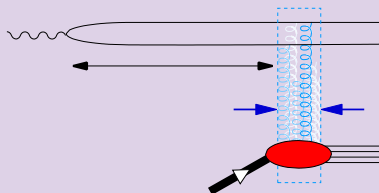
the photon splits



- target gluons Lorentz contracted to $\delta(x^-)$

Gluon production at increasing energy

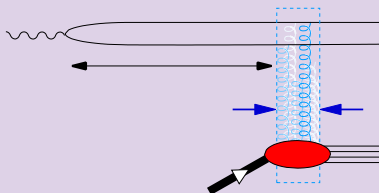
the photon splits



- target gluons Lorentz contracted to $\delta(x^-)$
- creation time dilated $\sim \frac{1}{x}$

Gluon production at increasing energy

the photon splits

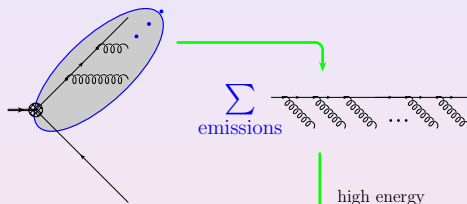


- target gluons Lorentz contracted to $\delta(x^-)$
- creation time dilated $\sim \frac{1}{x}$

$$\sum_{\text{gluons}} \text{[diagram of gluon lines]} = \text{P exp} -ig \int dz_\mu A^\mu(z)$$

$$= U_x$$

Eikonal factors arise due to high energies



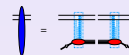
$$P \exp -ig \int dz_\mu A^\mu(z)$$

$U_{\mathbf{x}}$ @ small x (CGC)

$U_{\mathbf{p}}$ for jets

Cross section

$$\sigma_{\text{DIS}}(Y, Q^2) = \text{Im} \quad \gamma^* \quad \text{target}$$



Cross section

energy, $\ln 1/x$

$\sigma_{\text{DIS}}(Y, Q^2) = \text{Im} \int d^2r \, |\psi^2|(r^2 Q^2) \int d^2b \, \langle \frac{\text{tr}(1 - U_x U_y^\dagger)}{N_c} \rangle(Y)$

target

$r = x - y$

$b = (x + y)/2$

σ_{dipole}

Cross section

energy, $\ln 1/x$

$\sigma_{\text{DIS}}(Y, Q^2) = \text{Im} \int d^2r \psi^2(r^2, Q^2) \int d^2b \langle \frac{\text{tr}(1 - U_x U_y^\dagger)}{N_c} \rangle(Y)$

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$r = x - y$

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σ_{dipole}

- σ_{dipole} contains U_x

Cross section

energy, $\ln 1/x$

$\sigma_{\text{DIS}}(Y, Q^2) = \text{Im} \int d^2r \psi^2(r^2, Q^2) \int d^2b \langle \frac{\text{tr}(1 - U_x U_y^\dagger)}{N_c} \rangle(Y)$

σ_{dipole}

- σ_{dipole} contains U_x
- $\langle \dots \rangle(Y)$ hard!

Cross section

energy, $\ln 1/x$

$\sigma_{\text{DIS}}(\textcolor{red}{Y}, Q^2) = \text{Im} \int d^2\textcolor{green}{r} |\psi^2|(\textcolor{green}{r}^2 Q^2) \int d^2b \langle \frac{\text{tr}(1 - U_{\textcolor{blue}{x}} U_{\textcolor{blue}{y}}^\dagger)}{N_c} \rangle(\textcolor{red}{Y})$

target

$r = x - y$

$b = (x + y)/2$

σ_{dipole}

- σ_{dipole} contains $U_{\textcolor{blue}{x}}$

- $\langle \dots \rangle(\textcolor{red}{Y})$ hard!

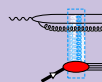
target wavefunction:
non-perturbative

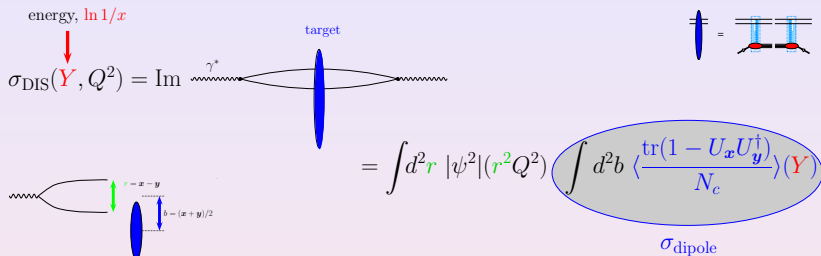
Cross section

The diagram illustrates the dipole picture of deep inelastic scattering (DIS). It shows a virtual photon γ^* interacting with a target (represented by a blue oval). The target is split into a quark and an antiquark, forming a dipole. The dipole is characterized by a size $r = x - y$ and a transverse position $b = (x + y)/2$. The dipole interacts with the target via a dipole cross-section σ_{dipole} . The diagram also shows the energy scale $\ln 1/x$ and the dipole cross-section $\sigma_{\text{DIS}}(Y, Q^2) = \text{Im} \int d^2r |\psi^2|(r^2 Q^2) \int d^2b \langle \frac{\text{tr}(1 - U_x U_y^\dagger)}{N_c} \rangle(Y)$.

- σ_{dipole} contains U_x
- $\langle \dots \rangle(Y)$ hard!

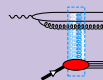
target wavefunction:
non-perturbative





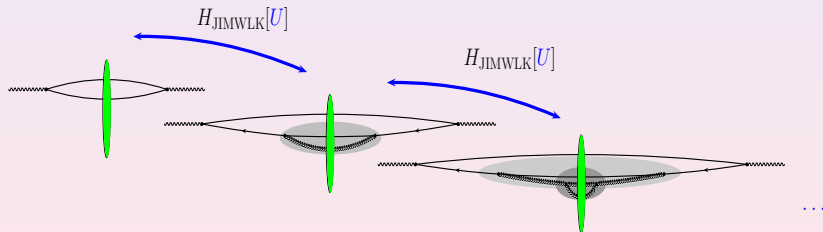
- $\langle \dots \rangle(Y)$ hard!

target wavefunction:
non-perturbative



- Bookkeeping device: $\langle \dots \rangle(Y) = \int \hat{D}[U] \dots \hat{Z}_Y[U]$

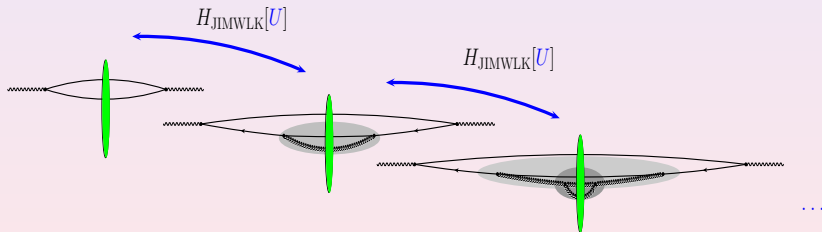
The JIMWLK evolution equation



The JIMWLK evolution equation

Heribert Weigert Nucl. Phys. A703, 2002, 823

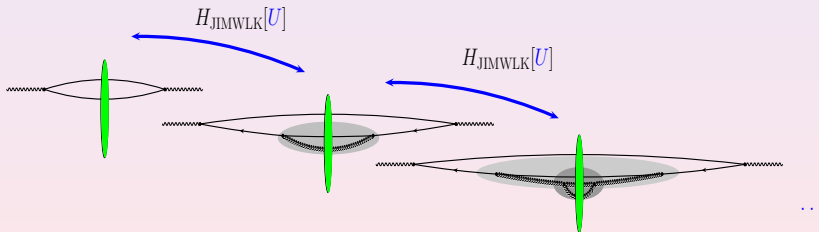
$$\frac{d}{d\mathbf{Y}} Z_{\mathbf{Y}}[\mathbf{U}] = -H_{\text{JIMWLK}}[\mathbf{U}] \quad Z_{\mathbf{Y}}[\mathbf{U}]$$



The JIMWLK evolution equation

Heribert Weigert Nucl. Phys. A703, 2002, 823

$$\frac{d}{d\mathbf{Y}} Z_{\mathbf{Y}}[\mathbf{U}] = -H_{\text{JIMWLK}}[\mathbf{U}] \quad Z_{\mathbf{Y}}[\mathbf{U}]$$



→ energy dependence of $\langle \dots \rangle(Y)$

The Balitsky hierarchy

- JIMWLK \longleftrightarrow infinite tower of **coupled** evol. eqns.

$$\partial_Y \langle \text{diagram} \rangle_Y = \frac{\alpha_s}{2\pi^2} \int d^2 z \tilde{\mathcal{K}}_{xyz} \langle \text{diagram}_1 - \text{diagram}_2 \rangle_Y$$

The diagram on the left is a horizontal line with a vertical green bar in the center, enclosed in a lens shape. The first diagram on the right is a horizontal line with a vertical green bar in the center, enclosed in a lens shape, with a shaded gray oval below the bar. The second diagram on the right is a horizontal line with a vertical green bar in the center, enclosed in a lens shape.

The Balitsky hierarchy

- JIMWLK \longleftrightarrow infinite tower of **coupled** evol. eqns.

$$\partial_Y \langle \text{diagram 1} \rangle_Y = \frac{\alpha_s}{2\pi^2} \int d^2 z \tilde{\mathcal{K}}_{xyz} \langle \text{diagram 2} - \text{diagram 3} \rangle_Y$$

$$\partial_Y \langle \text{diagram 4} \rangle_Y = \dots$$

The Balitsky hierarchy

- JIMWLK \longleftrightarrow infinite tower of **coupled** evol. eqns.

$$\partial_Y \left\langle \frac{\text{tr}(U_{\mathbf{x}} U_{\mathbf{y}}^\dagger)}{N_c} \right\rangle_Y = \frac{\alpha_s}{2\pi^2} \int d^2 z \tilde{\mathcal{K}}_{\mathbf{x}z\mathbf{y}} \left\langle \frac{[\tilde{U}_z]^{ab} 2 \text{tr}(t^a U_{\mathbf{x}} t^b U_{\mathbf{y}}^\dagger)}{N_c} - 2C_f \frac{\text{tr}(U_{\mathbf{x}} U_{\mathbf{y}}^\dagger)}{N_c} \right\rangle_Y$$

$$\partial_Y \left\langle \text{[diagram of a lens with a vertical green line]} \right\rangle_Y = \dots$$

The Balitsky hierarchy

- JIMWLK \longleftrightarrow infinite tower of **coupled** evol. eqns.

$$\partial_Y \left\langle \frac{\text{tr}(U_x U_y^\dagger)}{N_c} \right\rangle_Y = \frac{\alpha_s}{2\pi^2} \int d^2 z \tilde{\mathcal{K}}_{xyz} \left\langle \frac{[\tilde{U}_z]^{ab} 2 \text{tr}(t^a U_x t^b U_y^\dagger)}{N_c} - 2C_f \frac{\text{tr}(U_x U_y^\dagger)}{N_c} \right\rangle_Y$$

$$\partial_Y \left\langle \text{[diagram of a lens with a green vertical line]} \right\rangle_Y = \dots$$

- N_c limit factorizes: \longrightarrow single closed **nonlinear** eqn. (BK)

$$\partial_Y S_{xy} = \frac{\alpha_s N_c}{2\pi^2} \int d^2 z \tilde{\mathcal{K}}_{xyz} (S_{xz} S_{zy} - S_{xy}) \quad S_{xy} := \left\langle \frac{\text{tr}(U_x U_y^\dagger)}{N_c} \right\rangle$$

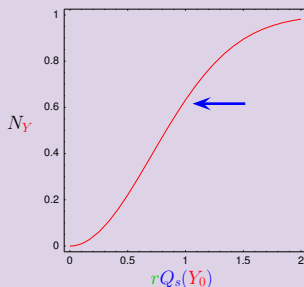
Saturation scale and cross section

$$\langle \dots \rangle(Y) \quad \rightarrow \quad \int d^2b \left\langle \frac{\text{tr}(1 - U_{\mathbf{r}} U_{\mathbf{0}}^\dagger)}{N_c} \right\rangle(Y) =: N_Y(\mathbf{r})$$

Saturation scale and cross section

$$\langle \dots \rangle(Y) \quad \longrightarrow \quad \int d^2b \left\langle \frac{\text{tr}(1 - U_{\mathbf{r}} U_{\mathbf{0}}^\dagger)}{N_c} \right\rangle(Y) =: N_Y(\mathbf{r})$$

Correlation length shrinks



$$R_s(Y) \sim \frac{1}{Q_s(Y)}$$

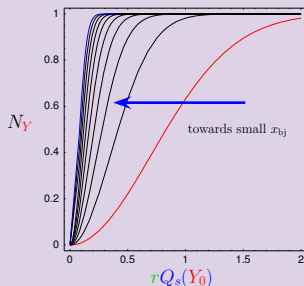
$R_s(Y) \equiv$ correlation length

$Q_s(Y) \equiv$ saturation scale

Saturation scale and cross section

$$\langle \dots \rangle(Y) \quad \longrightarrow \quad \int d^2b \left\langle \frac{\text{tr}(1 - U_{\mathbf{r}} U_{\mathbf{0}}^\dagger)}{N_c} \right\rangle(Y) =: N_Y(\mathbf{r})$$

Correlation length shrinks



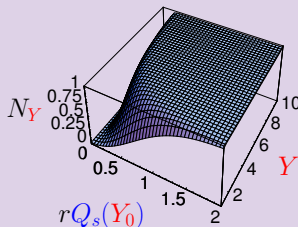
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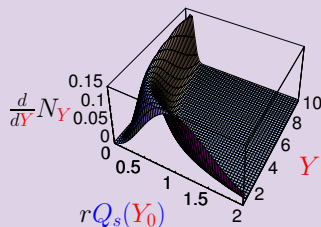
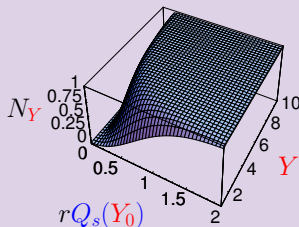
JIMWLK: IR safety and scaling

Activity above $\sim Q_s(Y)$



JIMWLK: IR safety and scaling

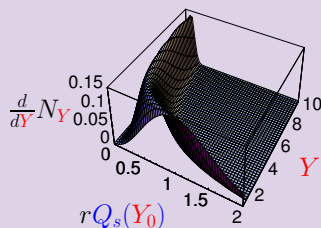
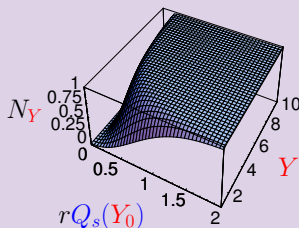
Activity above $\sim Q_s(Y)$



- activity follows $Q_s(Y)$

JIMWLK: IR safety and scaling

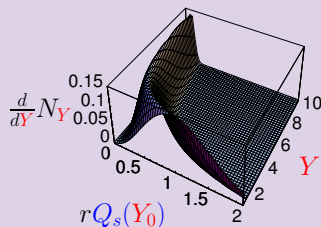
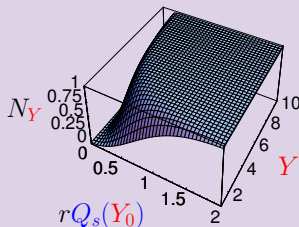
Activity above $\sim Q_s(Y)$



- activity follows $Q_s(Y)$
- IR safety

JIMWLK: IR safety and scaling

Activity above $\sim Q_s(Y)$

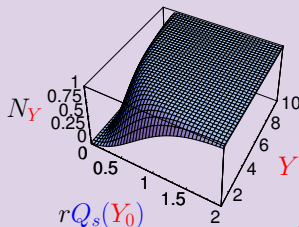


- activity follows $Q_s(Y)$
- IR safety
perturbative ✓

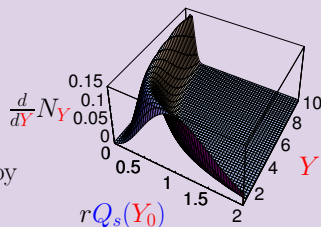
The saturation scale

JIMWLK: IR safety and scaling

Activity above $\sim Q_s(Y)$

scale \mathbf{r} by

$$\frac{Q_s(Y)}{Q_s(Y_0)}$$



- activity follows $Q_s(Y)$
 - IR safety
- perturbative ✓

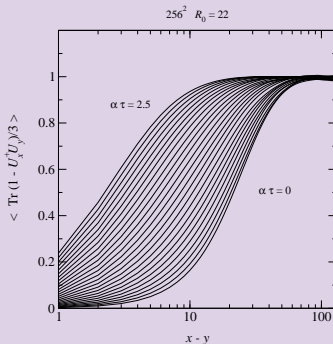
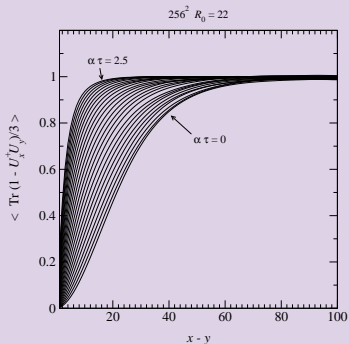
From simulations:

scaling with $Q_s(Y)$
[persists @ running coupling]

The saturation scale

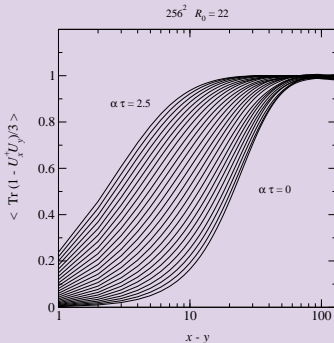
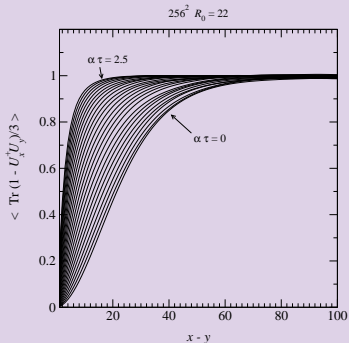
JIMWLK: simulations

Simulations show scaling (Rummukainen & H.W)



JIMWLK: simulations

Simulations show scaling (Rummukainen & H.W)

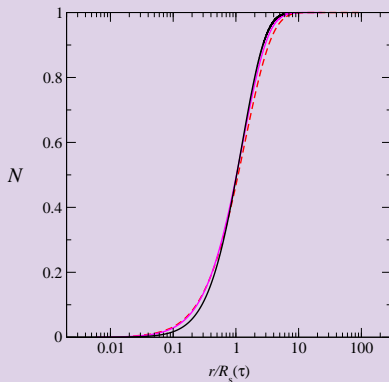


... in finite window: **lattice artefacts**

- $Q_s(Y)$ protects IR ✓
- check UV via $\lambda(Y) := \partial_Y \ln Q_s(Y)$

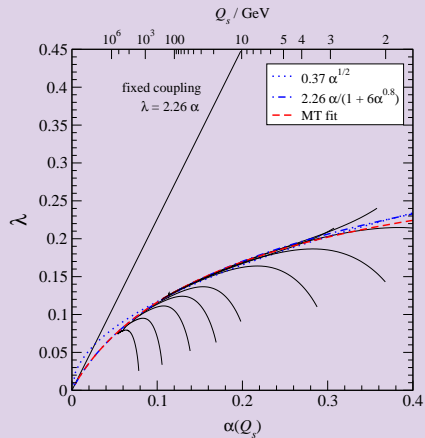
BK (parent dipole scheme): $\lambda(Y) = \partial_Y \ln Q_s(Y)$

Near scaling despite running coupling



► Running coupling is essential

$\lambda(Y) := \partial_Y \ln Q_s(Y)$

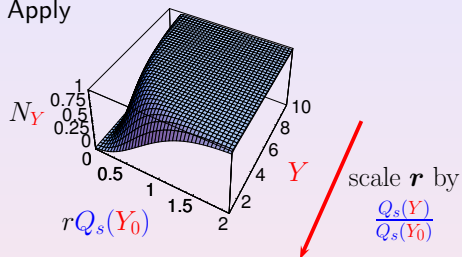


Outline

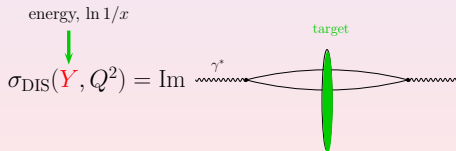
- 1 Motivation: gluons form the CGC
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 - Erasing the Cronin effect @ RHIC
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Geometric scaling @ HERA

Apply



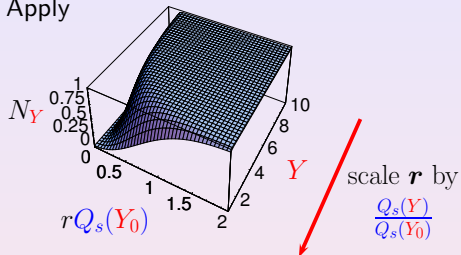
to Hera



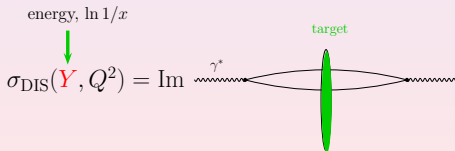
Golec-Biernat, Wüsthoff; PRD 60 (1999) 114023 [hep-ph/9903358]

Geometric scaling @ HERA

Apply



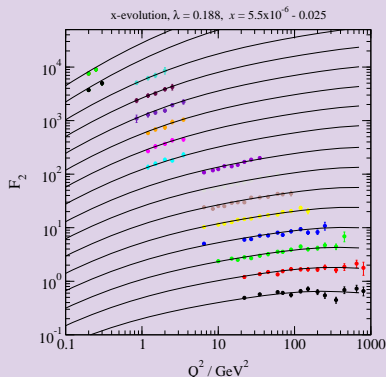
to Hera



Golec-Biernat, Wüsthoff; PRD 60 (1999) 114023 [hep-ph/9903358]

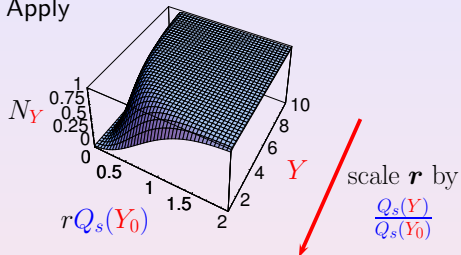
Phenomenological scaling fit to HERA:

$$\sigma(Y, Q^2) \sim F_2(Y, Q^2) \cdot Q^2$$

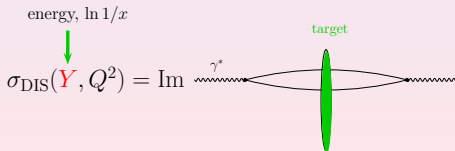


Geometric scaling @ HERA

Apply



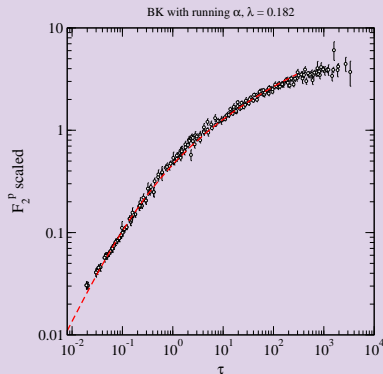
to Hera



Golec-Biernat, Wüsthoff; PRD 60 (1999) 114023 [hep-ph/9903358]

Phenomenological scaling fit to HERA:

$$\sigma(Y, Q^2) = \sigma(Y_0, \tau = Q^2 \frac{Q_s^2(Y_0)}{Q_s^2(Y)})$$



p Au: Cronin enhancement and evolution

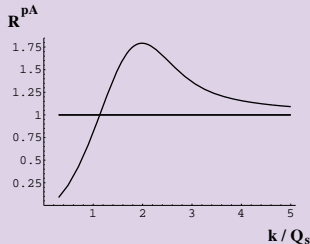
► A dep.

What is measured

$$R^{pA} = \frac{\frac{d\sigma^{pA}}{d^2k dY}}{A \frac{d\sigma^{pp}}{d^2k dY}}$$

Cronin effect

known from dilute systems
(fixed target experiments)



Evolution effects strongest where $Q_s(Y)$ largest

- at largest $Y \rightarrow$ most forward rapidities (small angles)
- most central collisions

p Au: Cronin enhancement and evolution

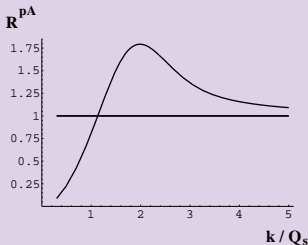
► A dep.

What is measured

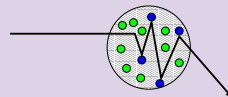
$$R^{pA} = \frac{\frac{d\sigma^{pA}}{d^2k dY}}{A \frac{d\sigma^{pp}}{d^2k dY}}$$

Cronin effect

known from dilute systems
(fixed target experiments)



Intuitive explanation



$$p_t \sim Q_s(Y)$$

Evolution effects strongest where $Q_s(Y)$ largest

- at largest $Y \rightarrow$ most forward rapidities (small angles)
- most central collisions

p Au: Cronin enhancement and evolution

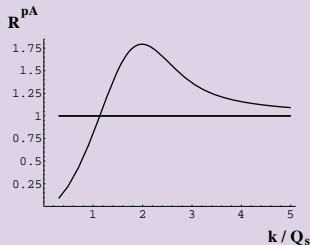
► A dep.

What is measured

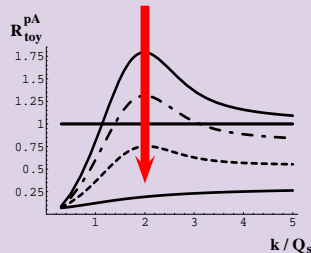
$$R^{pA} = \frac{\frac{d\sigma^{pA}}{d^2k dY}}{A \frac{d\sigma^{pp}}{d^2k dY}}$$

Cronin effect

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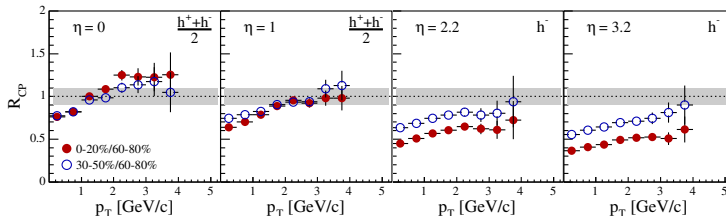
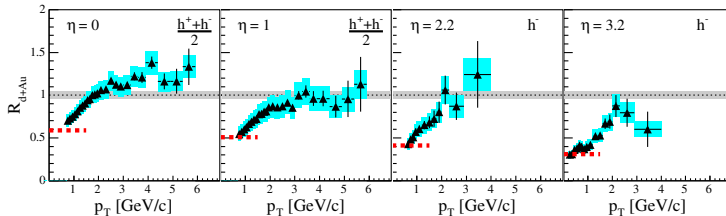
Effect of evolution



Evolution effects strongest where $Q_s(Y)$ largest

- at largest $Y \rightarrow$ most forward rapidities (small angles)
- most central collisions

Erasing the Cronin effect on the parton level [BRAHMS]



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Outlook: phenomena affected by the CGC

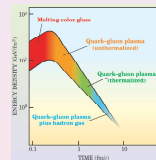
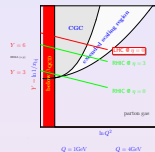
• CGC in γ^*A ✓

- saturation scale from JIMWLK ✓
- geometric scaling in eA ✓
- diffractive dissociation (rapidity gaps)
 - [Levin & Kovchegov; H.W. & Hentschinski]
- nonforward scattering, more differential observables

[more detailed info]

• CGC in heavy ion collisions (RHIC & LHC):

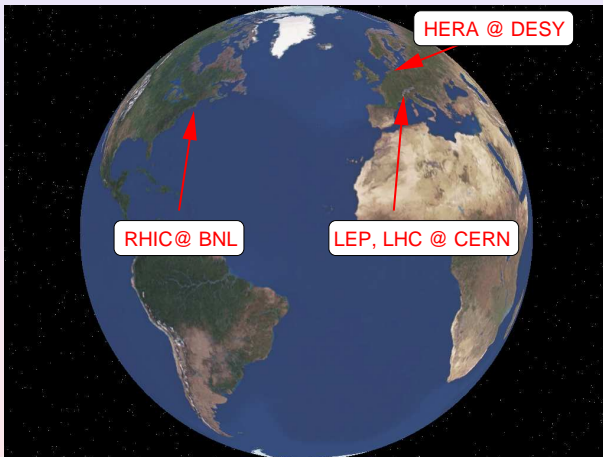
- saturation scale & Cronin effect ✓
- scales in initial conditions for QGP
- saturation scale & particle multiplicities



• jets in medium @ LHC

• cosmic ray showers [extremely high energies]

The Color Glass Condensate, a birds eye view



CGC in experiments @:

- RHIC, HERA
- LHC
- EIC (dedicated!)

Main characteristic:

- correlation length
 $R_s(Y) \sim \frac{1}{Q_s(Y)}$

Q_s -scaling:

- Y dependence
via $Q_s(Y)$

Outline

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The JIMWLK Hamiltonian

◀ back

$$H_{\text{JIMWLK}} = -\frac{1}{2} \frac{\alpha_s}{\pi^2} \kappa_{\mathbf{xzy}} \left[i \nabla_{\mathbf{x}}^a i \nabla_{\mathbf{y}}^a + i \bar{\nabla}_{\mathbf{x}}^a i \bar{\nabla}_{\mathbf{y}}^a + \tilde{U}_z^{ab} (i \bar{\nabla}_{\mathbf{x}}^a i \nabla_{\mathbf{y}}^b + i \nabla_{\mathbf{x}}^a i \bar{\nabla}_{\mathbf{y}}^b) \right]$$

$$\kappa_{\mathbf{xzy}} = \frac{(\mathbf{x} - \mathbf{z}) \cdot (\mathbf{z} - \mathbf{y})}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2}$$

[integration convention for x, z, y]

$i \nabla_{\mathbf{x}}^a$ and $i \bar{\nabla}_{\mathbf{x}}^a$ are functional derivatives:

$$i \nabla_{\mathbf{x}}^a := -[U_{\mathbf{x}} t^a]_{ji} \frac{\delta}{\delta U_{\mathbf{x},ij}}$$

$$i \bar{\nabla}_{\mathbf{x}}^a := [t^a U_{\mathbf{x}}]_{ji} \frac{\delta}{\delta U_{\mathbf{x},ij}}$$

The JIMWLK Hamiltonian

◀ back

$$H_{\text{JIMWLK}} = -\frac{1}{2} \frac{\alpha_s}{\pi^2} \mathcal{K}_{\mathbf{xzy}} \left[i \nabla_{\mathbf{x}}^a i \nabla_{\mathbf{y}}^a + i \bar{\nabla}_{\mathbf{x}}^a i \bar{\nabla}_{\mathbf{y}}^a + \tilde{U}_z^{ab} (i \bar{\nabla}_{\mathbf{x}}^a i \nabla_{\mathbf{y}}^b + i \nabla_{\mathbf{x}}^a i \bar{\nabla}_{\mathbf{y}}^b) \right]$$

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[integration convention for $\mathbf{x}, \mathbf{z}, \mathbf{y}$]

$i \nabla_{\mathbf{x}}^a$ and $i \bar{\nabla}_{\mathbf{x}}^a$ are functional derivatives:

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generate l. & r. inv vector fields, r & l rotations:

$$e^{-i\omega^a (i \nabla^a)} U = U e^{i\omega^a t^a} \quad e^{-i\omega^a (i \bar{\nabla}^a)} U = e^{-i\omega^a t^a} U$$

reps of the algebras:

$$[i \nabla^a, i \nabla^b] = i f^{abc} i \nabla^c \quad [i \bar{\nabla}^a, i \bar{\nabla}^b] = i f^{abc} i \bar{\nabla}^c \quad [i \bar{\nabla}^a, i \nabla^b] = 0$$

The JIMWLK Hamiltonian

◀ back

$$H_{\text{JIMWLK}} = -\frac{1}{2} \frac{\alpha_s}{\pi^2} \mathcal{K}_{\mathbf{xzy}} \left[i\nabla_{\mathbf{x}}^a i\nabla_{\mathbf{y}}^a + i\bar{\nabla}_{\mathbf{x}}^a i\bar{\nabla}_{\mathbf{y}}^a + \tilde{U}_z^{ab} (i\bar{\nabla}_{\mathbf{x}}^a i\nabla_{\mathbf{y}}^b + i\nabla_{\mathbf{x}}^a i\bar{\nabla}_{\mathbf{y}}^b) \right]$$

$$\mathcal{K}_{\mathbf{xzy}} = \frac{(\mathbf{x} - \mathbf{z}) \cdot (\mathbf{z} - \mathbf{y})}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2}$$


[integration convention for $\mathbf{x}, \mathbf{z}, \mathbf{y}$]

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$$i\bar{\nabla}_{\mathbf{x}}^a := [t^a U_{\mathbf{x}}]_{ji} \frac{\delta}{\delta U_{\mathbf{x},ij}}$$

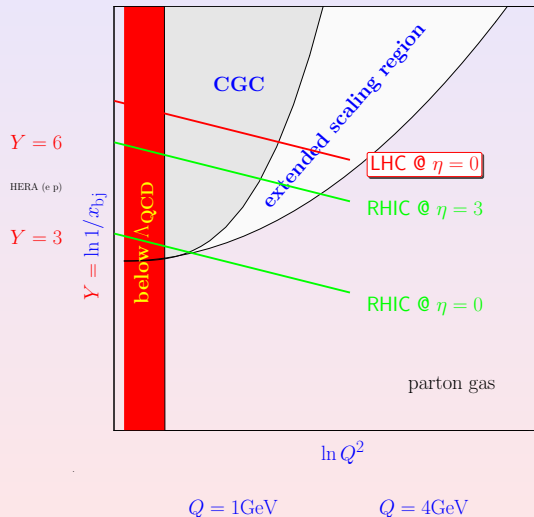
physics content:

- $\tilde{U}_z^{ab} (i\bar{\nabla}_{\mathbf{x}}^a i\nabla_{\mathbf{y}}^b + i\nabla_{\mathbf{x}}^a i\bar{\nabla}_{\mathbf{y}}^b)$ real emission
- $i\nabla_{\mathbf{x}}^a i\nabla_{\mathbf{y}}^a + i\bar{\nabla}_{\mathbf{x}}^a i\bar{\nabla}_{\mathbf{y}}^a$ virt. correction
- real emission term  nonlinear evolution

Outline

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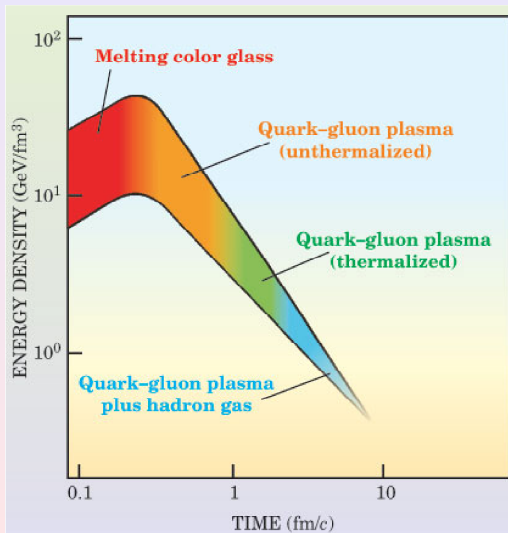
Colored Glass @ LHC with nuclei: YES



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From Colored Glass to Quark Gluon Plasma



McLerran, Ludlam
Physics Today
Oct 2003

← core of neutron star

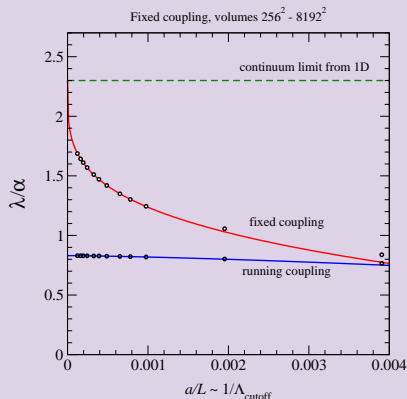
← nuclear matter

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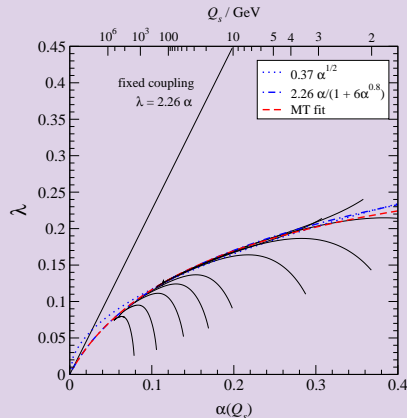
Running coupling is essential

phase space reduction



◀ back

$$\lambda(Y) := \partial_Y \ln Q_s(Y)$$



Outline

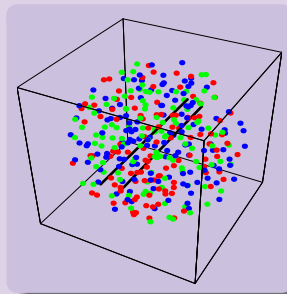
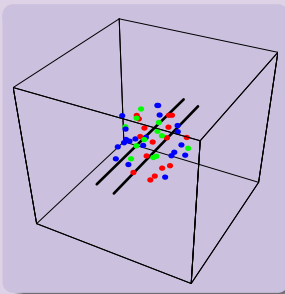
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A dependence of $Q_s(Y)$ from BK

◀ back

naive $A^{1/3}$ scaling

$$R \sim A^{1/3} \xrightarrow{\text{dilute}} (Q_s^A)^2 \sim (Q_s^p)^2 A^{1/3}$$



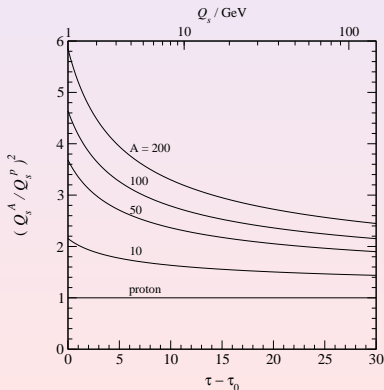
$$\text{dilute} \Leftrightarrow (Q_s^A)^2(Y_0) \sim (Q_s^p)^2 A^{1/3}(Y_0)$$

A dependence of $Q_s(Y)$ from BK

◀ back

rule of thumb estimate: $n \rightarrow 1/2$

$$\frac{Q_s^A(Y)^2}{\Lambda_{\text{QCD}}^2} = \exp \left\{ (n+1)2c \left(\frac{\pi}{\beta_0} \right)^n (Y - Y_0) + \left[\ln \left(\frac{A^{1/3} Q_s(Y_0)^2}{\Lambda_{\text{QCD}}^2} \right) \right]^{n+1} \right\}^{\frac{1}{n+1}}$$



- $\frac{Q_s^A(Y)^2}{Q_s^p(Y)^2} \xrightarrow{Y \rightarrow \infty} 1$
slowly

- $A = 50$:
13% loss
from $Y = 0 \dots 5$

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Non-global (**exclusive**) jet observables

[◀ back](#)

Already seen in $e^+e^- \rightarrow \text{jets}$ @ total energy E

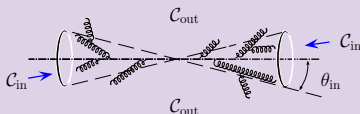


[◀ back](#)

Non-global (**exclusive**) jet observables

◀ back

Already seen in $e^+e^- \rightarrow \text{jets}$ @ total energy E

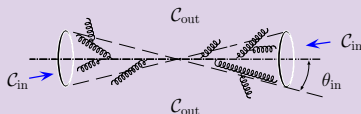


- fix geometry
- measure **soft** rad. into C_{out} **only**
- require $\sum E_{\text{soft}} < E_{\text{out}}$
- evolution equation in $\ln(E/E_{\text{out}})$

Non-global (**exclusive**) jet observables

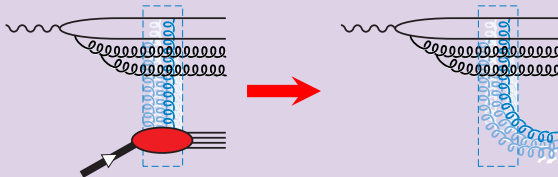
◀ back

Already seen in $e^+e^- \rightarrow \text{jets}$ @ total energy E



- fix geometry
- measure **soft** rad. into C_{out} **only**
- require $\sum E_{\text{soft}} < E_{\text{out}}$
- evolution equation in $\ln(E/E_{\text{out}})$

Analogy with CGC amplitudes:



[◀ back](#)

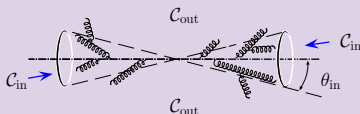
A diagram of a lens system. Two lenses are shown, separated by a distance d . The left lens has a focal length f_1 and the right lens has a focal length f_2 . An input beam with width C_{in} and angle θ_{in} enters from the left. The output beam has width C_{out} . The distance between the lenses is d .

- ## Of colored glass and saturation scales

Non-global (**exclusive**) jet observables

◀ back

Already seen in $e^+e^- \rightarrow \text{jets}$ @ total energy E



- fix geometry
- measure **soft** rad. into C_{out} **only**
- require $\sum E_{\text{soft}} < E_{\text{out}}$
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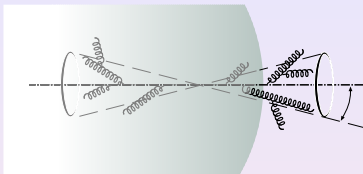
JIMWLK like evol. eqns. (BMS, H.W.)

$$\partial_{\ln(\frac{E}{E_{\text{out}}})} S_{pq} = \int_{C_{\text{in}}} \frac{d^2 \Omega_k}{4\pi} \bar{\alpha}_s w_{pq}(k) [S_{pk}(E) \cdot S_{kq}(E) - S_{pq}(E)]$$

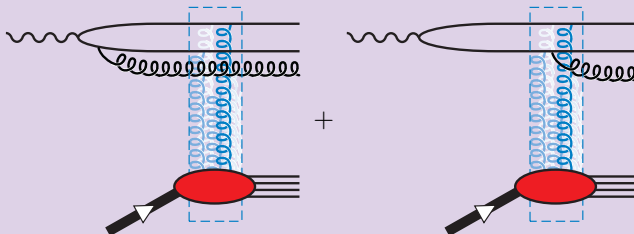
$$- \int_{C_{\text{out}}} \frac{d^2 \Omega_k}{4\pi} \bar{\alpha}_s w_{pq}(k) S_{pq}(E)$$

Jets in a medium

◀ back



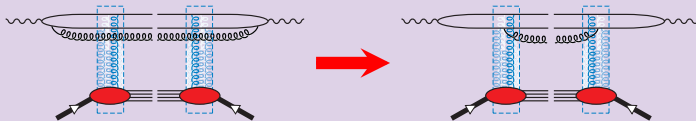
Already for global observables: difference to small x



Jets in a medium

◀ back

Evolution equations change, driven by



$$\int_{\tilde{C}_{\text{in}}} \frac{d^2 \Omega_k}{4\pi} w_{pq}(k) \frac{[\tilde{U}_k]^{ab} 2 \text{tr}(t^a U_p t^b U_q^\dagger)}{N_c} \rightarrow$$

$$\int_{\tilde{C}_{\text{in}}} \frac{d^2 \Omega_k}{4\pi} \int_0^\infty \frac{dx_0 dy_0}{p_0 q_0} e^{ip \cdot k x_0 / p_0 - iq \cdot k y_0 / q_0} p \cdot q (k^0)^2$$

$$\times \frac{[\tilde{U}_k]_{x_0, y_0}^{ab} 2 \text{tr}(t^a [U_p]_{x_0} t^b [U_q^\dagger]_{y_0})}{N_c} + \dots$$